Intro to Calc

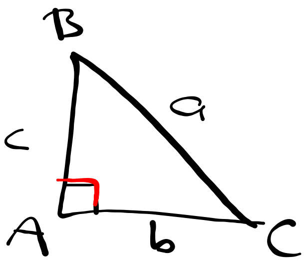
5.6 Law of Cosines

In any triangle with sides a,b and c and angles A,B, and C then the following equation is true:

SAS $a^2 = b^2 + c^2 - 2bc \cos A$

SSS $b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



$a^2 = b^2 + c^2 - 2bc\,\overbrace{\cos 90^\circ}^{0}$

$\underline{a^2 = b^2 + c^2}$. Pythagorean Theorem

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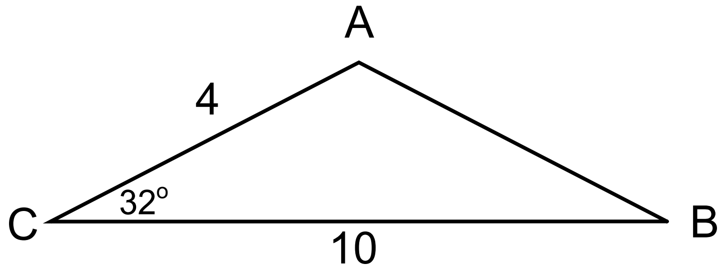
Ex. 1 SAS

Find all sides and angles of a triangle ABC given:



$a = 10$ $\quad A \approx 130.215°$

$b = 4$ $\quad B \approx 17.785°$

$c \approx 6.939$ $\quad C = 32°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 10^2 + 4^2 - 2(10)(4)\cos 32°$$

$$c^2 = 48.156\ldots$$

$$c \approx 6.939$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$B = \cos^{-1}\left(\frac{(10^2 + 6.939^2 - 4^2)}{(2(10)(6.939))}\right)$$

$$\approx 17.785°$$

$$A = 180 - 32 - 17.785$$

$$A \approx 130.215°$$

Using Law of Cosines solve for the Biggest angle first.

Using Law of sines solve for the SMALLEST angle first.

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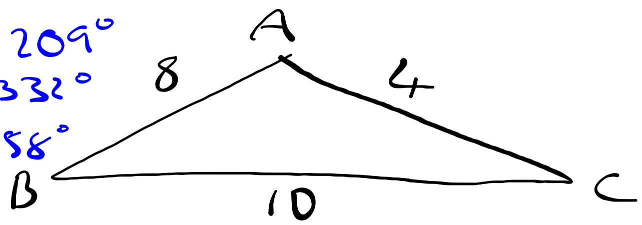
Ex. 2 SSS

Solve $\triangle ABC$ given that a=10, b=4 and c=8.

$a=10 \quad A \approx 108.209^\circ$

$b=4 \quad B \approx 22.332^\circ$

$c=8 \quad C \approx 49.458^\circ$



$$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

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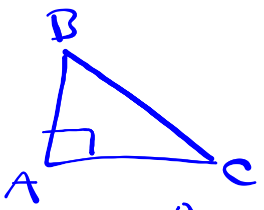
Triangle Area Formula

The general formula for finding the area of any triangle is:

$$\Delta Area = \frac{1}{2} bc \sin A$$

$= \frac{1}{2} ab \sin C$

$= \frac{1}{2} ac \sin B$



$A = \frac{1}{2} bc \sin 90^\circ$

$A = \frac{1}{2} bc$

Ex. 3

Find the area of the triangle with A=46°, b=24cm, c=16cm.

$A = \frac{1}{2} bc \sin A$

$= \frac{1}{2}(24)(16) \sin 46^\circ$

$\approx \underline{138.113} \text{ cm}^2$

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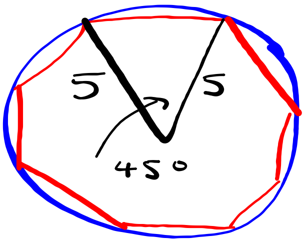
Ex. 4

Find the area of a regular octagon inscribed in a circle of radius 5 inches.

$\theta = \frac{360}{8}$

$\theta = 45°$



$A_{1\Delta} = \frac{1}{2}(5)(5)\sin 45°$

$= \frac{25\sqrt{2}}{4}$

$8\Delta =$ total area $= 8 \cdot \frac{25\sqrt{2}}{4}$

$= 50\sqrt{2}$

$\approx 70.711 \text{ in}^2$

The area of the octagon is $\approx 70.7 \text{ in}^2$

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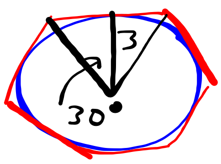
Ex. 5

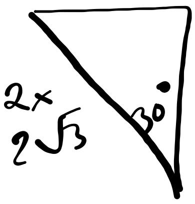
Find the area of a regular hexagon circumscribed about a circle of radius 3 inches.

$x = \sqrt{3}$

$\theta = \frac{360}{6}$

$= 60^\circ$





$3 = \sqrt{3}\,x$

$x = \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

12 little Δ's

$\text{Area of } \Delta = \frac{1}{2} bh$

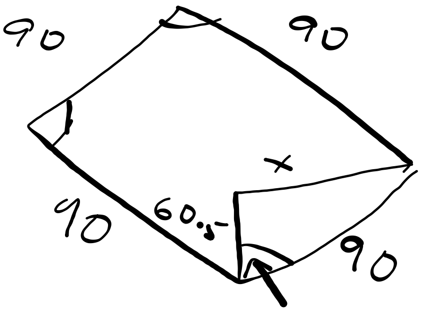
$= \frac{1}{2}\sqrt{3} \cdot 3$

$\text{total Area} = 12 \cdot \frac{1}{2} 3\sqrt{3}$

$= \underline{18\sqrt{3}}$

Ex. 6

The bases on a baseball diamond are 90ft apart, and the front edge of the pitcher's rubber is 60.5ft from the back corner of the plate. Find the distance from the center of the front edge of the pitcher's rubber to the far corner of the first base.



45°

$$x^2 = 90^2 + 60.5^2 - 2(60.5)(90)\cos 45°$$

$$x \approx 63.717 \text{ ft}.$$

The distance from the front of the pitcher's rubber to 1st base is $\approx$ 63.7 ft